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Superfluidity in the inner crust of neutron stars

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Abstract. We present a mean-field quantum calculation of the specific heat in the inner crust of neutron stars, taking into account the inhomogeneous character of the system, in which a lattice of neutron-rich nuclei coexists with a gas of unbound neutrons.

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1 Introduction

In the inner crust of neutron stars, that is, for baryon densities ranging between approximately 1.8×10^{-4} fm^{-3} and 0.1 fm^{-3}, matter consists of a Coulomb lattice of spherical neutron-rich nuclei, immersed in a sea of unbound neutrons and electrons. The system is expected to be superfluid, and this has important consequences on the thermal behaviour of the star. In the following, we discuss the results obtained from detailed quantum mean-field calculations of the pairing gap and of the specific heat [1,2], which take into account the spatial inhomogeneities introduced by the lattice.

2 Pairing gap in the Wigner-Seitz cell

One of the most detailed studies of the structure of the inner crust of neutron stars is the Hartree-Fock calculation of Negele and Vautherin [3], who determined the numbers of protons and of neutrons which are energetically favored at the different densities. More recent studies have essentially confirmed their results, which can be taken as a good starting point to study the superfluidity in the inner crust. Following the work of ref. [3], we have subdivided the inner crust in ten zones, which correspond to different values of the baryon density ρ, going from the deepest zone, $N_{\rm zone}=1$, corresponding to $\rho=1.3\times10^{14}$ g cm^{-3} (or 0.08 fm^{-3}), to $N_{\rm zone}=10$, corresponding to 4.7×10^{11} g cm^{-3} (or 3×10^{-4} fm^{-3}). Associated with each zone, there is a value for the radius $R_{\rm WS}$ of the Wigner-Seitz cell, namely the elementary cell of the Coulomb lattice. In each cell, part of the neutrons are bound to the nucleus placed at the center, while the remaining neutrons occupy orbitals at positive energies and their wave functions extend throughout the cell. The radial wave functions $\phi_{nlj}(r)$ in a given cell are obtained by solving the Schrödinger equation associated with a spherically symmetric Saxon-Woods potential, parametrized in such a way as to reproduce the density profiles inside the cell calculated in ref. [3]. We diagonalize the $^1S_0$ component of the v_{14} Argonne nucleon-nucleon potential in the (generalized) BCS approximation, in a basis composed of pairs of neutrons coupled to zero angular momentum and moving in states with n and n' number of nodes, taking into account the interplay of bound and unbound orbitals [1,2]. In fig. 1 we show the diagonal part of the pairing gap, Δ_{nnlj}, calculated at a density $\rho\approx0.02\,{\rm fm}^{-3}$ ($N_{\rm zone}=3$). We compare this result to a calculation performed in the same Wigner-Seitz cell, with the same number of particles but without the presence of the nucleus, that is, eliminating the Saxon-Woods potential and the bound single-particle states. It is seen from fig. 1 that close to the Fermi energy ($E_{\rm F}\approx13.5$ MeV in this case) the pairing gap is lower by a few hundred keV compared to the homogeneous case (neutron matter). In fact, a semiclassical estimate of the gap $\Delta_{nnlj}(E_{\rm F})$ is obtained [1] using the gap $\Delta_{\rm F}(k_{\rm F})$ calculated in neutron matter, shown in fig. 2: $\Delta_{nnlj}(E_{\rm F})$ is given by the volume average of the local pairing gap $\Delta_{\rm F}(k_{\rm F}(R))$ over the cell, where $k_{\rm F}(R)$ denotes the local Fermi momentum at a given point R in the cell. Inside the nucleus, the momentum is high and the gap is strongly suppressed (the nucleon-nucleon interaction becomes repulsive); this effect tends to decrease $\Delta_{nnlj}(E_{\rm F})$, and its

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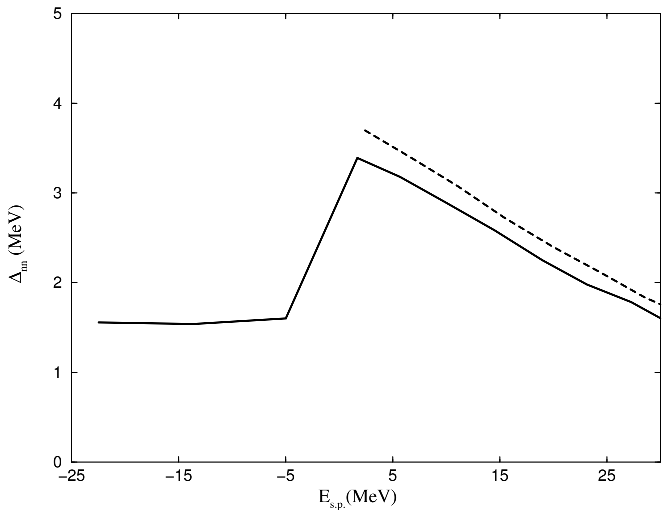

Fig. 1. Diagonal elements of the state-dependent pairing gap Δ_{nnlj} calculated using the Argonne interaction as a function of the single-particle energies (averaged over 5 MeV), with (solid line) and without (dashed line) the nucleus, in a Wigner-Seitz cell corresponding to a baryon density $\rho \approx 0.02$ fm^{-3}.

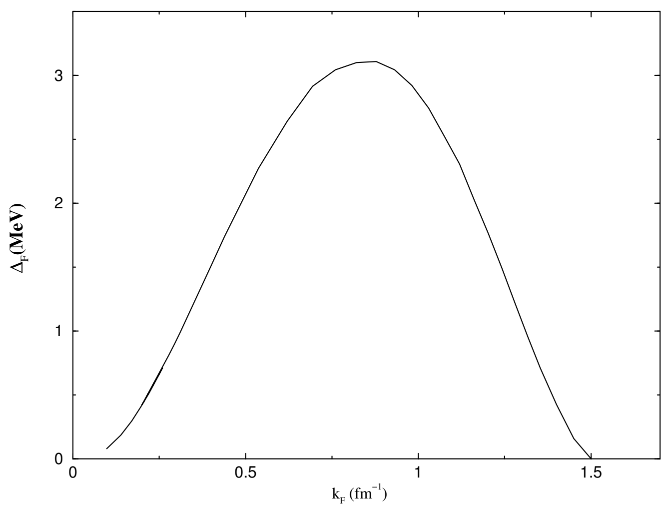

Fig. 2. The pairing gap at the Fermi energy, Δ_F, calculated in neutron matter with the Argonne interaction, is shown as a function of the Fermi momentum k_F.

importance depends on the portion of the volume of the cell occupied by the nucleus.

The specific heat of the inner crust is one of the basic properties which control the thermal behaviour of a neutron star after the early neutrino emission, which is expected to lower the temperature of the crust down to about $T = 0.1$ MeV. The specific heat depends exponentially on the pairing gap, $C_v \propto \exp(-\Delta/T)$, and it is shown in fig. 3 in the various zones of the crust. Its overall behaviour reflects the bell-shape dependence of the pairing gap in neutron matter as a function of density, or Fermi momentum, shown in fig. 2. The gap reaches its maximum around $k_F \approx 0.9$ fm^{-1}, a density which roughly corresponds to $N_{zone} = 3$, where the specific heat is minimum (cf. fig. 3). Approaching the surface of the star (for

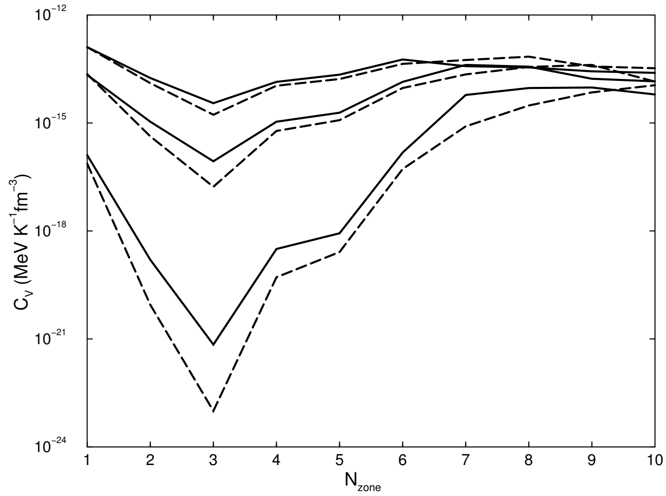

Fig. 3. Neutron specific heat calculated at three different temperatures $T = 0.1$ MeV (lower curve), 0.2 MeV and 0.3 (upper curve) in the different zones of the crust, with (solid line) and without (dashed line) the nucleus.

$N_{zone} > 3$), the specific heat increases, because the Fermi energy becomes smaller and the pairing gap decreases (in the present calculation the phase transition to a normal system takes place for $N_{zone} = 9$, or $\rho = 7 \times 10^{11}$ g cm^{-3}). The effect of the nucleus is particularly strong in the inner regions, essentially because the radius of the Wigner-Seitz cell is smaller there, and the nucleus occupies a larger fraction of its volume.

3 Conclusions

We have performed a mean-field quantum calculation of the pairing gap of the inner crust of neutron stars, taking into account its inhomogeneous character. Compared to the uniform case, the specific heat is lower by a factor 2-5, depending on the region of the crust. Whether this is relevant for the thermal evolution of the star, will depend on the model used to describe the cooling of the core of the star. Assuming a "rapid cooling" scenario, a detailed analysis [2] shows that cooling times may be affected by up to a factor of 2, depending on the temperature of the crust considered. The results of the quantum calculation presented here should also be important for realistic calculations of vortex pinning.

The present study should be considered as a first step towards a more complete investigation, which should take into account the induced interaction arising from polarization effects in the medium.

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